Christmas homework

Exercises:

1. A gambler plays the following game: she extract randomly a card from a deck of 40 cards (4 different seeds and 3 figures for each seed). She wins if she extract a hearts card or a ace. To play she has to pay 5€, and if she wins, she gains 30€.
   (a) If she plays 10 times (reinserting the extracted card in the deck each time), which is the probability that she will win more than 3 times?
   (b) Describe the random variable X that represent the total gain that the gambler can obtain (including the cost to play). Which is the mean and the variance of X?
   (c) Assume that she will play 1000 times, which is the probability that she will gain in total an amount between 500€and 1200€?

2. Assume the random variable X follows the distribution:
   $$f_X(x) = 3e^{-3x}, \quad x \geq 0$$
   Assume $Y = X^3$.
   (a) Which is the density function of Y?
   (b) Which is the covariance cov(X,Y)?
   (c) Which is the expected value of X and the expected value of Y? Do you notice any relation between them?

3. Consider the bivariate distribution
   $$f_{X,Y}(x, y) = kx(2 - y^2)$$
   for $1 < x < 2$ and $x < y < 2$
   (a) Which value do you have to assign to k in order to have a density function?
   (b) Which is the marginal density function of X?
   (c) Which is the marginal density function of Y?
   (d) Are the two marginals independent each other? Why?

4. Compute the following probabilities according to the definition of the random variable:
   (a) $X \sim N(2, 2), \quad P(X < 2.22)$
   (b) $X \sim N(-6, 30), \quad P(-10 < X < 1)$